Detecting Granular Time Series in Large Panels

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- This view has recently been challenged by a number of authors Gabaix (2011), Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015).
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- One of the main hurdles is that in many empirical applications it is often unclear which entities are granular
- In this paper we tackle this problem by
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  2. developing a granular detection methodology
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Roadmap

1. The granular detection problem and its applications
2. Granular model
   - Baseline Model
   - Identification
   - Inference
   - Extensions
3. Comparison with alternative approaches
4. Simulation Study
5. Applications:
   - Detecting granular sectors in industrial production
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Contribution

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  Meinshausen Buhlmann (2006); Peng, Wang, Zhou and Zhi (2009); Billio, Getmanksi, Lo and Pellizzon (2012); Diebold and Yilmaz (2014)

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The Granular Detection Problem

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- Let y_t be a n-dimensional time series generated as

$$\begin{aligned} y_{1:k,t} &= \boldsymbol{\Lambda}_1 f_t + g_t \\ y_{k+1:n,t} &= \boldsymbol{\Lambda}_2 f_t + \boldsymbol{\beta} g_t + \epsilon_t, \end{aligned}$$

  - f_t is an r-dim vector of common shocks with cov $\boldsymbol{\Sigma}_f$
  - g_t is a k-dim vector of granular shocks with cov $\boldsymbol{\Sigma}_g$
  - ϵ_t is an $(n-k)$-dim vector of non-granular shocks with cov $\boldsymbol{\Sigma}_\epsilon$
  - $\boldsymbol{\Lambda}_1$, $\boldsymbol{\Lambda}_2$ are $k \times r$ and $(n-k) \times r$ matrix of factor loadings
  - β is $n \times k$ matrix of granular loadings

- We call
  - $y_{1:k,t}$ the granular time series and
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  1. which series are granular and
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- Our objective consits in developing a statistical procedure to recover this information from the data

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Granular Detection and Factor Models

- Model has a factor representation but we do not propose a granular detection methodology based on factor techniques

- In this work the norms of the granular loadings do not diverge as n increases. Granulars are observed weak factors.

- This assumption is empirically reasonable.

- Simulations show that in this setting our methodology performs better than factor methods. Inline with Onatsky (2012) argument that factor analysis is more challenging when factors are weak.

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Motivating Examples

Granular sectors in industrial production

- Aggregate volatilility of US industrial production is large. This implies that variability of individual sectors does not average out.

- Two leading explainations for this phenomenon:
  1. Aggregate shocks – Forster *et al.* (2011)
  2. Sector specific shocks – Gabaix (2011), Acemoglu *et al.* (2012)

- The granular model with common factors applied to the panel of sectoral industrial production can disentangle both explanations and our methodology can be applied to detect which sectors influence aggregate fluctuations

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Granular institutions in the financial system

- One of the lessons learnt from the Great Financial Crisis is that the distress of few yet highly influential financial firms can impose significant negative externalities on the entire the economy
- The GFC has motivated a large literature that aims at detecting and ranking systemic institutions in the financial system according to their "systemicness"
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Granular Model

Model

Granular Model (without common factors)

- Let y_t be a n-dimensional time series generated as

$$\begin{aligned} y_{1:k,t} &= g_t \\ y_{k+1:n,t} &= \boldsymbol{\beta} g_t + \epsilon_t, \end{aligned}$$

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  - β is $n \times k$ matrix of granular coefficients

Heuristics

Heuristics

- Assume for simplicity that $\|\beta_i\| > 1$ and that $\Sigma_\epsilon = \mathsf{I}$
- The concentration $\mathsf{K} = \Sigma^{-1}$ of y_t is

$$\mathsf{K} = \begin{bmatrix} \Sigma_g^{-1} + \beta'\beta & -\beta' \\ -\beta & \mathsf{I} \end{bmatrix}$$

- Notice that the columns/rows of K matrix corresponding to the non-granular series are sparse
- This motivates us to develop a granular detection strategy based on the column (or rows) norms of K, denoted by

$$\|\mathsf{K}_i\|$$

In this example the column norms corresponding to granulars are larger than the colum norms of the non-granulars

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Network Interpretation

- Granular detection parameter has a network interpretation
- Classic network/graphical representation for y_t is the so called partial correlation network
- The panel y_t is represented by a graph $\mathcal{G}$ defined over n vertices
  - each component of y_t is associated with a vertex
  - y_{it} and y_{jt} are connected by an egde iff the partial correlation ρ^{ij} between y_{it} and y_{jt} is nonzero

- Partial correlations are related to $\mathbf{K}$: $\rho^{ij} \neq 0$ iff $k_{ij} \neq 0$

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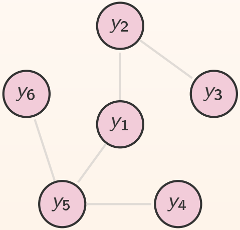

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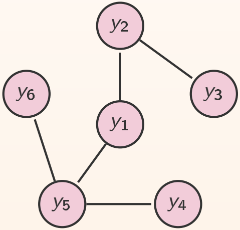

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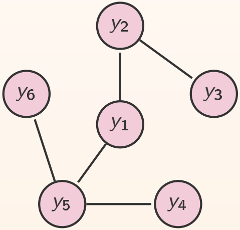

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Refresher on Partial Correlation

- Partial Correlation measures (cross-sect.) linear conditional dependence between $y_{t\,i}$ and $y_{t\,j}$ given on all other variables:

$$\rho^{ij} = \mathrm{Cor}(y_{t\,i}, y_{t\,j} | \{y_{t\,k} : k \neq i, j\}) = -\frac{k_{ij}}{\sqrt{k_{ii} k_{jj}}}.$$

- Partial Correlation is related to Linear Regression: For instance, consider the model

$$y_{1\,t} = c + \beta_{1\,2} y_{2\,t} + \beta_{1\,3} y_{3\,t} + \beta_{1\,4} y_{4\,t} + \beta_{1\,5} y_{5\,t} + u_{1\,t}$$

β_{13} is different from 0 $\Leftrightarrow$ 1 and 3 are partially correlated

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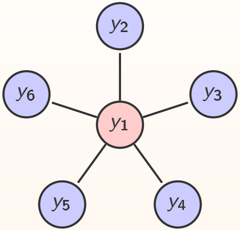
- In our example, if there are $n = 6$ series and $k = 1$ granulars, then the concentration $\mathbf{K}$ and the partial correlation network $\mathcal{G}$ are

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & 0 & 0 & 0 & 0 \\ k_{31} & 0 & k_{33} & 0 & 0 & 0 \\ k_{41} & 0 & 0 & k_{44} & 0 & 0 \\ k_{51} & 0 & 0 & 0 & k_{55} & 0 \\ k_{61} & 0 & 0 & 0 & 0 & k_{66} \end{bmatrix}$$



- Note that $\|\mathbf{K}_i\|$ is proportional to the number of connections of vertex i and granulars can be thought as hubs in the partial correlation network representation of y_t

- Granualar model has analogies with partial correlation network model with power law structure

Barigozzi, Brownlees, Lugosi (2017)

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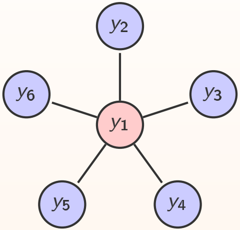

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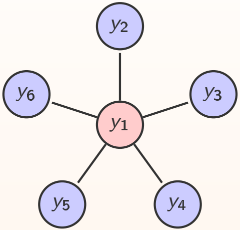

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- Granualar model has analogies with partial correlation network model with power law structure

  Barigozzi, Brownlees, Lugosi (2017)

Difference with Factor Models

- Consider $n = 6$ series and $r = 1$ common factor,

$$y_t = \boldsymbol{\lambda} f_t + u_t$$

with $\mathrm{Var}(f_t) = \boldsymbol{I}_r$ and $\mathrm{Var}(u_t) = \boldsymbol{I}_n$

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with $c = -\frac{1}{1+\lambda'\lambda}$

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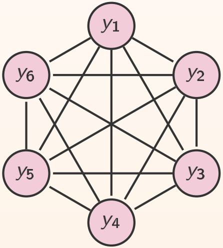

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Detecting Granulars when Factors are Present

- Consider the granular panel data model with factors

$$\begin{aligned} y_{1:k,t} &= \Lambda_1 f_t + g_t \\ y_{k+1:n,t} &= \Lambda_2 f_t + \beta g_t + \epsilon_t \end{aligned}$$

- Then

$$\Sigma^{-1} = K - K\Lambda(\Sigma_f + \Lambda' K \Lambda)^{-1}\Lambda' K$$

where K is the concentration matrix of the model without factors

- Remarks
  - The common factors impose a low-rank perturbation of the concentration matrix of the simple granular model captured by K
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Relation with the Network Literature

- There is a large literature on networks estimation that typically estimates the network by constructing sparse a estimators of $\mathbf{K}$
- In order to assess which series are more influential in the network, the typically procedure consits of counting the number of edges of each vertex, that is for series i the number of nonzero $\hat{k}_{ij}$.
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Granular Detection

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- The heuristic example suggests the column norms of the concentration matrix allow to detect granular series in the panel

- Our granular detection strategy is based on:
  1. Granular Ranking: We rank the series in the panel on the basis of the norm of the columns of the sample concentration matrix

$$||\hat{K}_i||$$

  2. Granular Selection: We choose the number of granulars by maximising the sequential column ratio statistic

$$\hat{k} = \underset{s=1,\ldots,n-1}{\text{argmax}} \; ||\hat{K}_{(s)}|| \,/\, ||\hat{K}_{(s+1)}||$$

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Assumptions

Assumption 1

1. $\mathrm{E}(g_t) = 0$ and $\mathrm{E}(g_t g_t') = \Sigma_g$ with $\Sigma_g \succ 0$.
2. $\mathrm{E}(\epsilon_t) = 0$ and $\mathrm{E}(\epsilon_t \epsilon_t') = \Sigma_\epsilon$ with $\Sigma_\epsilon \succ 0$.
3. $\mathrm{E}(g_t \epsilon_{i,t}) = 0$ for all i, t
4. We have that $\boldsymbol{\beta}'\boldsymbol{\beta} \to \boldsymbol{D}$ as $n \to \infty$, with $\mu_k(\boldsymbol{D}) > 0$ and $\mu_1(\boldsymbol{D}) < \infty$. Also, there exists an integer $N > 0$ such that for all $n > N$ the columns of $\boldsymbol{\beta}$, denoted by $\boldsymbol{\beta}_i$ for $i = 1, \ldots, k$, satisfy

$$||\boldsymbol{\beta}_i|| > \kappa_\beta \kappa_\epsilon$$

where κ_ϵ and κ_β are the condition number of Σ_ϵ and $\boldsymbol{D}$.

Remarks

- Granular shocks have a pervasive effect on the entire panel: The larger β the more pervasive the granular shocks
- Assumption 1 (iv) is key for identification
- Special cases:
  - If $k = 1$ then $||\beta|| > \kappa_\epsilon$
  - If $\beta'\beta \to C_\beta I$ then $C_\beta > \kappa_\epsilon$
- Stronger versions of this assumption (Assumption 1 (iv*)) required to identify k

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Granular Ranking Identification Lemma

Lemma 2

Let y_t be generated by the granular model (without common factors) under assumption 1 (i)-(iv).

Then we have for $n > N$ that

$$\|K_i\| > \|K_j\| \qquad \text{for all} \quad i = 1, \ldots, k, \quad \text{and} \quad k+1, \ldots, n,$$

where K_l denotes the l-th column of K.

Granular Selection Identification Lemma

Lemma 3

Let y_t be generated by the granular model (without common factors) under assumption 1 (i)-(iii) and (iv*).

Then we have for $n > N^*$, when $k > 0$ that

$$k = \underset{s=1,\ldots,n-1}{\operatorname{argmax}} \; \|K_s\| \; / \; \|K_{s+1}\|$$

where K_l denotes the l-th column of K.

Estimation

Estimation

- We operationalize our identification lemmas by using the column norm of the sample concentration matrix

$$||\hat{K}_i||$$

which is estimated using on the inverse of the sample covariance matrix

Assumptions

Assumption 2

Let y_t be an n-dimensional time series process.

1. $\{y_t\}$ is stationary and ergodic.
2. $\{y_t\}$ is α-mixing. There exists positive constants γ_1 and C_1 such that for all positive integers t we have that the α mixing coefficients satisfy

$$\alpha(t) \leq \exp(-C_1 t^{-\gamma_1}) \ .$$

3. There exists positive constants γ_2 and C_2 such that for any x with $\|x\| = 1$ and $s > 0$ and any $i = 1, ..., n$

$$\Pr(|x'\Sigma^{-1/2} y_t| > s) \leq \exp(-(s/C_2)^{\gamma_2}) \ .$$

4. Let γ be defined as $\gamma^{-1} = \gamma_1^{-1} + 2\gamma_2^{-1}$. Then, $\gamma < 1$.

Assumptions: Remarks

- Set of assumption common in the large dimensional covariance estimation literature

  Fan, Lia and Mincheva (2012)

- These set of assumptions allow to use a Bernstein-type inequality for mixing data established by Merlevede, Peligrad and Rio (2011) which is key to establish the consistency results of the paper

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Estimation

Theorem 1: Consistency

Let y_t be generated by the granular model (without factors) under assumptions 1 (i)-(iv), 2 and 3. Suppose $n \to \infty$ and $T = O(n^{2/\gamma - 1})$. Then, for any $\eta > 0$ there exists positive constants $C_1, \ldots, C_5$ such that

1. $\mu_n(\Sigma) - C_1\sqrt{\frac{n}{T}} \leq \mu_n(\hat{\Sigma}) \leq \mu_1(\hat{\Sigma}_y) \leq \mu_1(\Sigma) + C_2\sqrt{\frac{n}{T}}$
2. $\left\| \hat{\Sigma} - \Sigma \right\| \leq C_3\sqrt{\frac{n}{T}}$
3. $\left\| \hat{K} - K \right\| \leq C_4\sqrt{\frac{n}{T}}$
4. $\|\hat{K}_i\| - \|K_i\| \leq C_5\sqrt{\frac{n}{T}}$

at least with probability $1 - O(n^{-\eta})$.

Estimation

Corollary: Consistent Granular Detection

Let y_t be generated by model under 2 (i)-(ii) and 1 (i)-(iv). Consider the event $\mathcal{E}_R$ and $\mathcal{E}_S$ defined as

$$\mathcal{E}_R = \left\{ \|\hat{K}_i\| > \|\hat{K}_j\| \text{ for all } \ i = 1, ..., k \text{ and } j = k+1, ..., n \right\}$$
$$\mathcal{E}_S = \left\{ \hat{k} = k \right\}$$

Then,

$$\mathrm{Pr}(\mathcal{E}_R) \geq 1 - \mathcal{O}\left(n^{-\eta}\right) \text{ and } \mathrm{Pr}(\mathcal{E}_S) \geq 1 - \mathcal{O}\left(n^{-\eta}\right)$$

Extensions

Granular Model with Common Factors

- Empirically, the vast majority of economic and financial panels exhibit evidence of a factor structure. It is straightforward to extend our methodology to the case in which the series in the panel are influenced by a set of common factors.

- Let y_t be a n-dimensional time series generated as

$$\begin{aligned} y_{1:k,t} &= \Lambda_1 f_t + g_t \\ y_{k+1:n,t} &= \Lambda_2 f_t + \beta g_t + \epsilon_t , \end{aligned}$$

  - f_t is a r-dimensional vector of common factors
  - g_t is a k-dimensional vector of granular shocks
  - ϵ_t is an $n-k$-dimensional vector of non-granular shocks
  - Λ_1 and Λ_2 are $k \times r$ and $(n-k) \times r$ matrices of factor loadings
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Granular Model with Common Factors

- We extend the identification and estimation results to the granular models with factors
- In this setting:
  1. Granular detection strategy is unaltered: we rank and select granulars on the basis of the column norms of K
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Comparison with other methods

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- Granular detection has analogies with the problem of detecting observed factors.

  Stock and Watson (2002), Bai and Ng (2006), Parker and Sul (2016)

- Roughly, these methods typically consist of
  1. Estimating the number of factors
  2. Choosing as observed factors series whose R^2 is the largest when regressed on estimated factors

- These methods are not designed specifically for the class of models we work in this paper. However, we compare our methodology with this approach. We provide examples of granular models in which factor do not work satisfactorily. We also compare the approaches in the simulation study.

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Simulation Study

- We carry out a simulation study to the properties of the granular detection methodology
- We consider a granular model with factors

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y_{1:k,t} &= \Lambda_1 f_t + g_t \\
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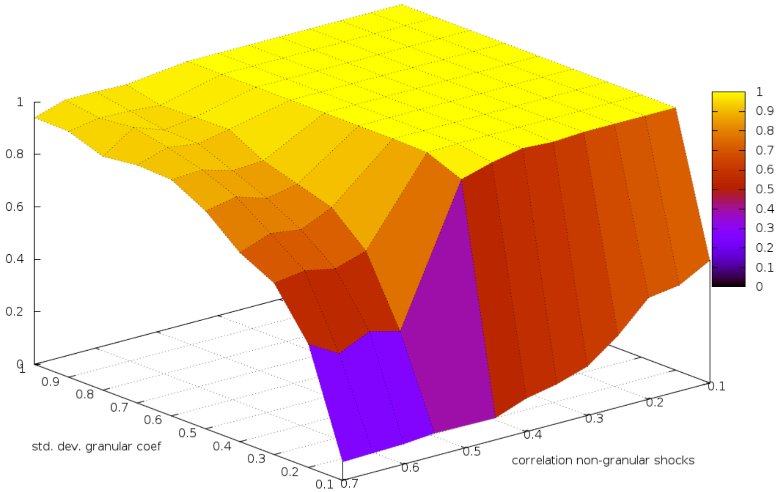
Granular Ranking Probabilities

				σ_b						
n	T	k	r	0.01	0.05	0.10	0.25	0.50	0.75	1.00
50	200	3	0	0.140	0.800	0.967	1.000	1.000	1.000	1.000
100	200	3	0	0.140	0.926	0.996	1.000	1.000	1.000	1.000
50	400	3	0	0.207	0.941	0.998	1.000	1.000	1.000	1.000
100	400	3	0	0.266	0.992	1.000	1.000	1.000	1.000	1.000
50	200	5	0	0.206	0.813	0.954	0.997	1.000	1.000	1.000
100	200	5	0	0.252	0.932	0.997	1.000	1.000	1.000	1.000
50	400	5	0	0.187	0.905	0.992	1.000	1.000	1.000	1.000
100	400	5	0	0.334	0.991	1.000	1.000	1.000	1.000	1.000
50	200	3	5	0.103	0.676	0.876	0.969	0.983	0.985	0.989
100	200	3	5	0.148	0.865	0.972	0.998	0.998	0.998	0.999
50	400	3	5	0.104	0.800	0.953	0.989	0.992	0.994	0.994
100	400	3	5	0.184	0.967	0.996	0.999	1.000	0.999	1.000
50	200	5	5	0.156	0.721	0.884	0.970	0.988	0.988	0.993
100	200	5	5	0.216	0.878	0.976	0.998	0.999	0.998	0.999
50	400	5	5	0.150	0.822	0.950	0.990	0.995	0.997	0.998
100	400	5	5	0.273	0.966	0.997	1.000	0.999	0.999	1.000

Granular Selection Probabilities

				σ_b						
n	T	k	r	0.01	0.05	0.10	0.25	0.50	0.75	1.00
50	200	3	0	0.098	0.190	0.520	0.883	0.979	0.992	0.994
100	200	3	0	0.104	0.377	0.773	0.969	0.995	0.999	1.000
50	400	3	0	0.093	0.404	0.833	0.988	0.998	1.000	1.000
100	400	3	0	0.090	0.732	0.974	0.999	1.000	1.000	1.000
50	200	5	0	0.044	0.101	0.357	0.832	0.959	0.969	0.986
100	200	5	0	0.048	0.265	0.782	0.981	0.995	0.998	0.999
50	400	5	0	0.042	0.207	0.661	0.963	0.993	0.997	0.999
100	400	5	0	0.047	0.652	0.941	0.998	1.000	1.000	1.000
50	200	3	5	0.117	0.136	0.310	0.482	0.594	0.587	0.615
100	200	3	5	0.115	0.273	0.556	0.771	0.789	0.781	0.795
50	400	3	5	0.098	0.188	0.418	0.656	0.704	0.708	0.723
100	400	3	5	0.135	0.525	0.785	0.882	0.884	0.875	0.878
50	200	5	5	0.036	0.068	0.141	0.428	0.575	0.626	0.641
100	200	5	5	0.042	0.159	0.507	0.784	0.825	0.838	0.852
50	400	5	5	0.044	0.105	0.296	0.608	0.708	0.746	0.759
100	400	5	5	0.047	0.425	0.787	0.902	0.916	0.922	0.917

Granular Detection: σ_b vs c



Comparison with Factor-based methods

				σ_b						
n	T	k	r	0.01	0.05	0.10	0.25	0.50	0.75	1.00
50	200	3	0	0.062	0.582	0.835	0.977	0.993	0.995	0.995
100	200	3	0	0.000	0.561	0.860	0.991	0.997	0.999	0.999
50	400	3	0	0.056	0.701	0.949	0.999	1.000	1.000	1.000
100	400	3	0	0.006	0.855	0.993	1.000	1.000	1.000	1.000
50	200	5	0	0.044	0.636	0.843	0.954	0.977	0.979	0.976
100	200	5	0	0.036	0.655	0.899	0.992	0.997	0.998	0.999
50	400	5	0	0.000	0.618	0.874	0.981	0.992	0.992	0.994
100	400	5	0	0.002	0.754	0.966	0.999	1.000	1.000	1.000
50	200	3	5	0.419	0.331	0.544	0.830	0.935	0.952	0.954
100	200	3	5	0.153	0.218	0.591	0.938	0.991	0.999	0.996
50	400	3	5	0.365	0.299	0.554	0.876	0.963	0.980	0.979
100	400	3	5	0.136	0.201	0.655	0.983	0.999	1.000	1.000
50	200	5	5	0.446	0.347	0.502	0.736	0.859	0.884	0.891
100	200	5	5	0.207	0.250	0.536	0.904	0.983	0.991	0.992
50	400	5	5	0.461	0.312	0.492	0.762	0.899	0.936	0.942
100	400	5	5	0.160	0.242	0.604	0.967	0.998	1.000	0.999

Empirical Applications

Granular sectors in industrial production

Granular sectors in industrial production

- We consider a panel of US industrial production series ($n = 138$) during from 1985 until 2007 ($T = 276$)

  Foerster, Sarte & Watson (2011)

- Substantial interest in trade-off between idiosyncratic and aggregate shocks, e.g. Long and Plosser (1987)

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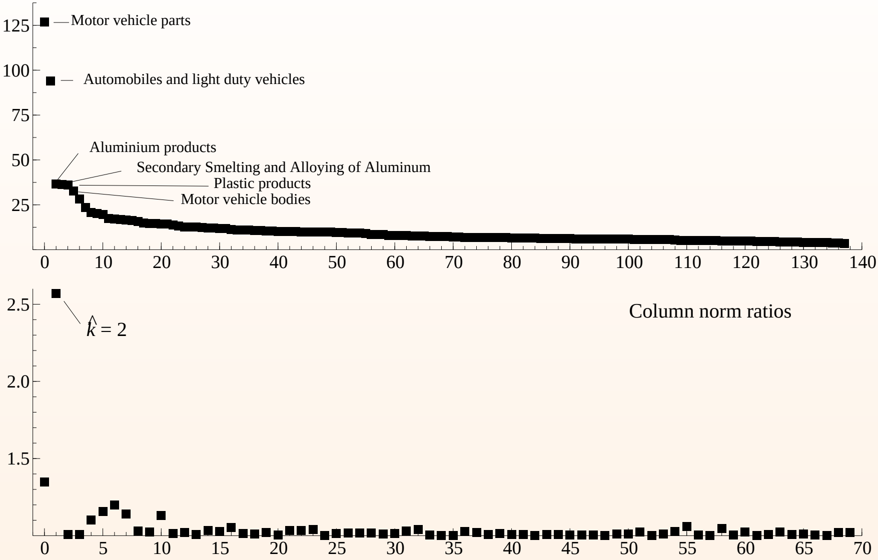

Detecting Granular Time Series in Large Panels

Column norm based ranking	
Sector	$\|\hat{K}_{(i)}\|$
Motor Vehicle Parts	126.76
Automobiles and Light Duty Motor Vehicles	94.02
Aluminum Extruded Products	36.58
Plastics Products	36.30
Miscellaneous Aluminum Materials	35.99
Motor Vehicle Bodies	32.70
Paper and Paperboard Mills	28.26
Household and Institutional Furniture and Kitchen Cabinets	23.59
Commercial and Service Industry Machines	20.70
Motor Homes	20.07

Detecting Granular Time Series in Large Panels

PCA based R^2 ranking	
Sector	R^2
Plastics Products*	0.651
Household and Institutional Furniture and Kitchen Cabinets*	0.520
Metal Valves Except Ball and Roller Bearings	0.476
Architectural and Structural Metal Products	0.448
Other Miscellaneous Manufacturing	0.441
Sawmills and Wood Preservation	0.429
Reconstituted Wood Products	0.423
Fabricated Metals: Forging and Stamping	0.423
Fabricated Metals: Spring and Wire Products	0.422
Commercial and Service Industry Mach/Other Gen Purpose Mach	0.406

Detecting Granular Time Series in Large Panels

- We find $\hat{k} = 2$ granulars (Motor vehicle parts & Automobiles)
- We find $\hat{r} = 1$ common factors
- We estimate the model with

$$\begin{bmatrix} f_t \\ g_t \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} f_{t-1} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}$$

- Compute impulse response of 1 sd shock to granular sector (controlling for common factors)

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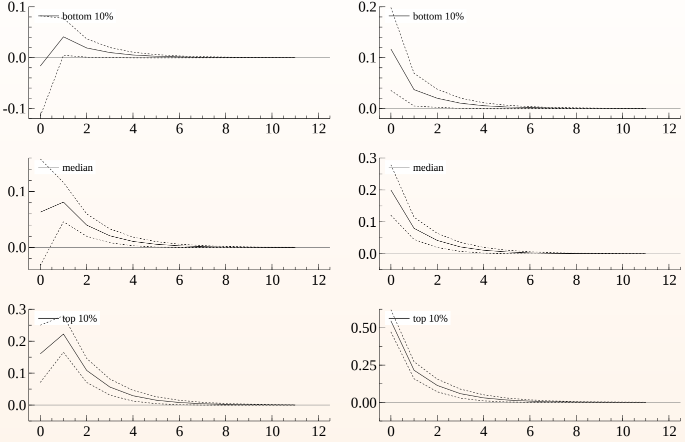
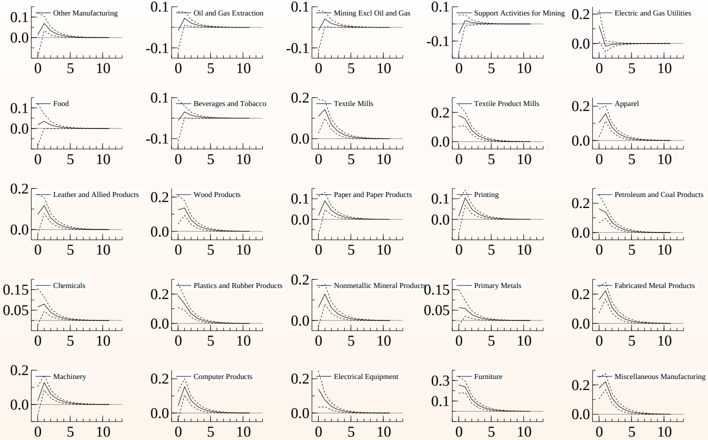
Granular IR



We show the response of a 1-standard deviation shock to the granular series (left column) and common factor (right column) on the 14th largest, the median and the 124th largest non-granular industrial production series.

Granular IR



Impulse responses industrial production. We show the response of a 1-standard deviation shock to the granular series on the industrial production series that corresponds to the within sector median according to the loadings β.

Granular firms in the financial system

Granular firms in the financial system

- We consider a panel of large US financial firms ($n = 88$) during the 2007-2009 Great Financial Crisis ($T = 503$)

  Diebold & Yilmaz (2014), Brownlees & Engle (2016) and Acharya, Pedersen, Philippon & Richardson (2016)

- Volatility is measured using the high-low range

$$\tilde{\sigma}^2_{i,t} = 0.361 \left(p^{high}_{i,t} - p^{low}_{i,t} \right)^2$$

and hence $y_{i,t} = \tilde{\sigma}^2_{i,t}$

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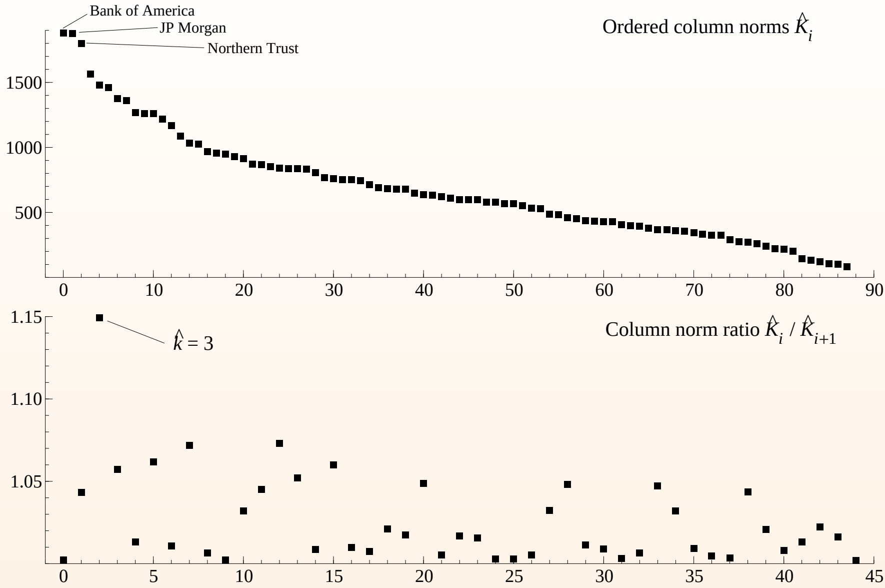

Granular firms in the financial system

Rank	Granulars	K-Ratio	G-SIB	D-SIB
1	Bank of America	5.209	✓	
2	JPMorgan	11.555	✓	
3	Northern Trust	16.466		✓
4	Associated Banc-Corp	2.567		
5	Wells Fargo	1.897	✓	
6	Comerica	6.154		✓
7	Torchmark	0.204		
8	Goldman Sachs Group	1.648	✓	
9	Cullen/Frost Bankers	2.192		
10	SunTrust Banks	3.332		✓
11	KeyCorp	1.479		✓
12	Bank of Hawaii	0.744		
13	Commerce Bancshares/MO	0.531		
14	Allstate	1.916		
15	U.S. Bancorp	4.345		✓
16	Valley National Bancorp	3.884		
17	Regions Financial	2.368		✓
18	Morgan Stanley	3.877	✓	
19	PNC Financial	2.957		✓
20	Principal Financial Group	2.549		

Conclusions

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- In this paper we propose a model that formalizes the notion of granular time series and we introduce a granular detection methodology
- Methodology inspired by the network literature in econometrics and statistics
- We establish the large sample properties of our procedure
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